Application of Mathematics in Modeling and Analysis of Geophysical Phenomena

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Abstract: *Geophysical phenomena exhibit highly nonlinear, multiscale, and multiphysics coupling characteristics, and their modeling and analysis heavily rely on the rigorous structure of mathematical methods and the adaptability of computational strategies. With the growth of observational data and the increasing complexity of modeling requirements, geophysical models are transitioning from analytical expressions to high-dimensional numerical simulations integrated with intelligent algorithms. Focusing on the critical role of mathematics in geophysical modeling, this study systematically explores the mathematical structural characteristics of geophysical systems, the modeling mechanisms of typical phenomena, and the applicable pathways of tools such as multiscale analysis, high-order computation, and machine learning in data processing. The results show that mathematics is not only the core language for characterizing the behavior of Earth systems but also an important bridge connecting physical processes, observational data, and predictive mechanisms, providing solid support for the modeling and understanding of complex natural systems.*

Keywords: *Geophysical modeling; Partial differential equations; Inverse problems; Multiscale analysis; High-order numerical methods; Integration of mathematics and machine learning*

Introduction

Geophysical phenomena broadly encompass various natural processes, including atmospheric circulation, seismic wave propagation, and geomagnetic field inversion, which essentially exhibit highly nonlinear, dynamically coupled, and multiscale interactive system behaviors. In fields such as global change, resource exploration, and disaster monitoring, the capability to model and predict these processes has become a critical technological bottleneck. Mathematics, as a fundamental tool for constructing theoretical models, achieving numerical representation, and guiding inversion analysis, plays an irreplaceable role in geophysical research. Particularly under complex boundary conditions, uncertain inputs, and massive observational data, traditional physical models alone can no longer meet the dual requirements of modeling accuracy and computational efficiency. It is therefore necessary to reconstruct the mathematical modeling system by integrating high-order numerical algorithms, data-driven strategies, and physical prior constraints, thereby promoting a paradigm shift in modeling logic from “analytical–numerical” to a bidirectional integration of “physics–data.” Based on typical geophysical phenomena, this study establishes a research framework from theoretical abstraction to tool adaptation, aiming to reveal the intrinsic coupling relationship between mathematical language and the Earth system, and to provide theoretical support and methodological insights for subsequent model optimization and algorithmic innovation.

1. Theoretical Basis of Mathematical Modeling in Understanding Geophysical Phenomena

1.1 Mathematical Structural Characteristics of Geophysical Systems

Geophysical systems exhibit significant nonlinear, multiscale, and strongly coupled characteristics, with their operating mechanisms involving the interactions of multiple physical fields, such as gravity, magnetic, stress, and temperature fields. The dynamic evolution of these field variables in space and time constitutes the fundamental processes of material transport and energy diffusion in continuous media. Mathematically, such complex coupled systems are generally described by systems of partial differential equations, whose core structures are characterized by nested combinations of nonlinear convection–diffusion terms, anisotropic tensor coefficients, and boundary control terms. Taking atmospheric circulation and mantle convection as examples, their evolution equation systems possess strong nonlinearity, mixed boundary conditions, and ill-posedness, which significantly increase the complexity of model analysis and numerical processing [1].

In addition, geophysical phenomena generally involve cross-scale behaviors ranging from microscopic scales (e.g., pore fluid flow, seismic fracture propagation) to macroscopic scales (e.g., plate motion, climate system regulation). Such multilevel coupling imposes high requirements on system integration and variable coordination in mathematical modeling. In model structures, the boundary interactions and energy conversion relationships among multiple physical fields need to be expressed mathematically through appropriate coupling mechanisms, such as conformal mapping, variational principles, or generalized constitutive relations, to maintain both physical consistency and mathematical controllability in modeling. Therefore, geophysical modeling is not merely a quantitative description of natural phenomena but rather a product of the synergistic interaction among mathematical structure design, physical principle embedding, and system behavior prediction.

1.2 Evolutionary Logic of Mathematical Modeling Paradigms in Geophysics

The evolution of geophysical modeling paradigms reflects a staged transition from traditional analytical models to high-precision numerical models and further to intelligent data-driven models. This process embodies not only the updating of mathematical tools but also a paradigm shift in modeling logic from “theory-dominated” to “theory–computation–data integration.” In early geophysical modeling, researchers primarily relied on simplified equation forms and analytical solutions under specific boundary conditions, such as the application of Laplace and Poisson equations in gravity and static magnetic field modeling. However, when confronted with practical problems involving complex geological structures and dynamic boundaries, the limitations of analytical methods gradually became apparent. This led to the emergence of high-order numerical discretization techniques, including finite difference, finite element, and spectral methods, to solve geophysical models involving irregular grids, heterogeneous media, and dynamic boundaries.

At present, with the exponential growth of geophysical observation data and the development of machine learning algorithms, geophysical modeling is progressively entering a new stage driven by “model–data coupling.” In this stage, mathematical modeling not only addresses system dynamic equations and boundary value problems but also integrates intelligent algorithm modules, such as data assimilation, feature extraction, pattern recognition, and probabilistic inference, to achieve adaptive parameter correction and dynamic optimization of predictive capabilities. The evolution of this modeling logic requires mathematical models to possess greater robustness and scalability, supporting responses to uncertainty, incompleteness, and non-stationarity in their representational forms. For example, in seismic inversion problems, traditional regularization methods have gradually been integrated with data-driven techniques such as sparse representation and low-rank approximation, forming multilayered model frameworks that combine physical constraints with data adaptability.

1.3 Synergistic Mechanism between Mathematical Rationality and Physical Reasonability in the Modeling Process

The effective construction of geophysical models must achieve deep integration between mathematical structures and physical laws, namely, expressing the essential logic of physical reasonability through the formalism of mathematical rationality. Mathematical rationality is reflected in the formal consistency of the model, the controllability of its solution space, and the guarantee of convergence, whereas physical reasonability requires the model to comply with fundamental principles such as energy conservation, momentum transfer, and mass balance. The key to the synergistic mechanism lies in accurately transforming boundary conditions, initial states, and dynamic evolution pathways of physical processes into mathematical language while ensuring consistency in terms of continuity, stability, and solvability. For example, in heat conduction problems, a mathematical model must not only capture the spatial distribution of temperature gradients but also represent thermal response characteristics through the functional expression of the material’s thermal conductivity. In seismic wave propagation, the construction of wave equations must fully account for the anisotropy and heterogeneity of subsurface media while maintaining energy closure under boundary reflection conditions.

The construction of this synergistic mechanism also involves the fusion processing of multisource heterogeneous data and the closed-loop optimization of modeling feedback pathways. In practical applications, physical laws are often embedded in model structures in the form of parameters, and the acquisition of these parameters relies on observational data and numerical inversion processes. At this point, mathematical optimization methods must establish a bridge between data constraints and physical mechanisms, such as by employing Bayesian inversion, Kalman filtering, and Tikhonov regularization to achieve iterative parameter updates and uncertainty control. Moreover, when dealing with complex model structures, the tension between physical interpretability and mathematical parsimony must also be balanced to avoid system divergence or parameter unidentifiability caused by excessive model degrees of freedom. Therefore, the essence of geophysical mathematical modeling is not merely a stacking of technical means but rather an integrated construction process that fuses physical ontology with mathematical formal logic, with its effectiveness rooted in the bidirectional reconciliation of rational expression and natural laws [2].

2. Mechanisms for Constructing Mathematical Models of Typical Geophysical Phenomena

2.1 Partial Differential Equation Modeling in Atmospheric and Oceanic Circulation

Atmospheric and oceanic circulation systems, as typical geophysical multiphase flow systems, are essentially governed by systems of partial differential equations derived from fundamental principles such as mass conservation, momentum conservation, and energy conservation. The core of such models lies in the coupled representation of the Navier–Stokes equations, continuity equations, and heat transfer equations, which describe the evolution of velocity, pressure, temperature, and density fields of fluids to capture the physical essence of large-scale flows. Particularly at the global scale, the Coriolis force, gravitational potential, and turbulent viscosity become indispensable terms in the models, forming a highly nonlinear and tightly coupled mathematical system. Considering Earth’s curvature, rotation, and stratified structure, it is also necessary to introduce multiscale terms in spherical coordinate systems and free-surface perturbation conditions to ensure that the models more closely reflect realistic circulation characteristics.

To address the complex boundaries and nonstationary disturbances in atmospheric and oceanic systems, high-precision spatial discretization and time integration methods are required. Finite difference and finite volume methods exhibit good performance in terms of conservation and grid adaptability, while spectral methods demonstrate high efficiency in simulating large-scale flows. Parameterization techniques are often introduced in model construction to handle subgrid-scale processes, such as statistical approximations of turbulence, convection, and radiative transfer. Such models rely on mathematical analysis to identify instability mechanisms and impose strict error control requirements on initial-value sensitivity and chaotic behavior, having evolved into partial differential equation–based computational platforms with predictive, analytical, and regulatory capabilities.

2.2 Mathematical Representation Framework of Seismic Wave Propagation Models

The propagation of seismic waves is a dynamic process in which elastic disturbances travel in wave form through subsurface media, and its mathematical modeling is fundamentally based on elastic wave equations. These equations originate from the generalized forms of Newton’s second law and Hooke’s law in continuous media, typically expressed either as velocity–stress coupled systems or displacement-based wave equations. In isotropic media, the propagation of P-waves and S-waves can be described by scalar and vector wave equations, respectively, whereas in anisotropic and layered media, tensorial stiffness matrices must be introduced to characterize directional dependencies. Parameters in the wave equations, such as density and elastic moduli, determine the propagation characteristics of the medium and its interface reflection behavior, and their heterogeneity directly affects the propagation paths and energy distribution of the wavefield [3].

At the mathematical modeling level, seismic wave propagation is characterized by the coupling of initial–boundary value problems and high-frequency source terms, and the models face challenges such as singular disturbances, non-smooth boundaries, and strong nonlinear material responses. To capture fine-scale structures and interface effects during wave propagation, numerical solutions require high-order spatial discretization techniques and adaptive mesh refinement strategies, such as staggered-grid finite difference methods, high-order finite element methods, and spectral element methods, to achieve high-fidelity simulation of wavefield morphology and amplitude evolution. Meanwhile, to represent interface scattering, dispersion effects, and dissipation characteristics under realistic geological conditions, fractional derivative terms, viscoelastic constitutive relations, or viscous damping terms can be introduced, thereby extending the model's capability to characterize complex dynamic behaviors. Wave propagation modeling not only provides theoretical support for source mechanism analysis, fault structure characterization, and medium parameter inversion but also establishes a mathematical foundation for vibration response prediction and engineering seismic simulations.

2.3 Mathematical Inverse Problem Construction for Gravity and Magnetic Field Inversion

The inversion of Earth's gravity and magnetic fields essentially falls within the category of typical inverse problem modeling, namely, inferring subsurface physical parameters or source-body structures from spatial observation data. Unlike forward problems, inverse problems are generally ill-posed, characterized by non-uniqueness, instability, and incomplete data. Mathematically, such problems are often expressed in the form of Fredholm integral equations or convolutions, where source-body parameters are related to observational data through integral transformations involving Green's functions. The key to constructing inverse problem models lies in establishing reasonable prior constraints and regularization mechanisms to enhance model stability and physical interpretability.

Inverse problem modeling is typically based on least-squares objective functions, combined with constraints such as Tikhonov regularization, L1 norms, or total variation to construct solvable optimization models. For large-scale or nonlinear problems, efficient parameter inversion can be achieved using methods such as the conjugate gradient method, quasi-Newton method, or variational Bayesian approaches. Gravity and magnetic field inversion are widely applied to density structure identification and structural anomaly detection. To improve resolution, strategies such as multiscale regularization, model compression, and deep learning-assisted reconstruction can be introduced, integrating traditional modeling with modern data-driven methods to build robust, high-precision inversion systems, thereby strengthening the mapping capability between the Earth's field and subsurface structures [4].

3. Mathematical Tools in Geophysical Data Analysis: Adaptability and Optimization Strategies

3.1 Application of Multiscale Analysis Methods in Nonstationary Data Processing

Geophysical observation data generally exhibit complex characteristics such as nonstationarity, nonlinearity, and coexistence of multiple frequencies, and traditional Fourier analysis has significant limitations in capturing transient variations and local disturbances. Multiscale analysis methods, by constructing time–frequency localized representations, effectively adapt to the dynamic structural variations of nonstationary geophysical signals. Among these methods, wavelet transform, with its multiresolution characteristics, plays an important role in signal decomposition, mutation detection, and boundary identification. It is particularly suitable for layered analysis and local feature extraction of seismic signals, geoelectric resistivity profiles, and geomagnetic disturbance data. The customizability of wavelet basis functions allows for the preservation of essential structural information in different frequency bands while suppressing noise interference, thereby enhancing the representational capacity and physical interpretability of model input data.

Adaptive methods such as the Hilbert–Huang transform (HHT) and empirical mode decomposition (EMD) provide more flexible analysis channels for strongly nonlinear and nonstationary data. These methods do not require predefined basis functions but instead extract intrinsic mode functions (IMFs) in a data-driven manner, enabling dynamic tracking of local frequency characteristics. They have been widely applied in source identification, crustal response analysis, and extreme event monitoring. To overcome the limitations of EMD in mode mixing and endpoint effects, improved algorithms such as ensemble empirical mode decomposition (EEMD) and complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) can be introduced to enhance decomposition stability and improve the interpretability of components. Multiscale analysis methods represent a modeling shift from static frequency-domain analysis to dynamic structural-domain analysis in geophysical data processing, serving as a critical bridge between mathematical tools and the physical mechanisms underlying observational information.

3.2 High-Order Computational Methods and Accuracy Control Mechanisms in Numerical Simulation

Partial differential equations in geophysical models typically involve strong nonlinearity, anisotropy, and multiscale coupling terms, and standard low-order numerical methods face bottlenecks in balancing solution accuracy and computational efficiency. High-order computational methods, by introducing higher-order interpolation, derivative approximation, and mesh reconstruction strategies, significantly enhance local accuracy and global convergence while maintaining computational stability. Spectral methods, as a representative high-order numerical technique, approximate the solution space with global function expansions and are particularly suitable for wave problems in periodic boundaries and smooth domains, such as atmospheric circulation, seismic wave propagation, and electromagnetic field modeling. Their exponential convergence characteristics provide a considerable accuracy advantage in high-resolution simulations [5].

To ensure the applicability of numerical solutions under conditions such as boundary treatment, medium discontinuity, and geometric complexity, high-order finite element methods, weighted essentially non-oscillatory (WENO) schemes, and spectral element methods (SEM) have become mainstream approaches. These methods achieve local enhancement in target regions through adaptive mesh refinement, unstructured mesh generation, and error estimation mechanisms, thereby improving the resolution of complex local structures such as fault zones, abrupt topographies, and heterogeneous medium interfaces. In practical simulations, accuracy control mechanisms ensure the reliability of numerical results by monitoring residuals, evaluating error propagation, and adjusting time steps, while adaptive evolution strategies dynamically allocate computational resources to improve overall simulation efficiency. High-order computational methods constitute the core mathematical foundation of modern geophysical numerical simulation platforms, and their accuracy control capability is directly related to the reliability of simulation predictions and the completeness of structural reconstruction.

3.3 Integration Pathways between Machine Learning Methods and Traditional Mathematical Models

When addressing modeling requirements involving high-dimensional observational data, nonlinear responses, and complex coupled structures, the expressive capability of traditional mathematical models becomes increasingly limited. Machine learning methods, with their self-learning, adaptive, and nonparametric modeling capabilities, provide new pathways for geophysical modeling and analysis. Supervised-learning-based inversion neural networks can learn nonlinear functional relationships between seismic records and subsurface structural parameters, demonstrating high efficiency in fault identification and source mechanism inference. Unsupervised learning methods, such as clustering analysis and autoencoder networks, can identify intrinsic patterns in multisource geophysical data, revealing regional structural evolution and spatial anomaly distribution. Deep learning methods, in particular, have significant application value in three-dimensional geophysical image recognition, subsurface structure reconstruction, and wavefield prediction, forming an efficient mapping chain from raw data to structural models [6].

The core of the integration pathway lies in ensuring physical consistency and mathematical stability by embedding machine learning within the physical modeling framework to construct hybrid models that combine data-driven and mechanism-constrained components. Introducing physical loss functions and boundary conditions during model training helps enhance the interpretability and physical plausibility of the results. Machine learning can also serve as a tool for initial-value or boundary-value correction in numerical models, improving simulation stability and convergence. Generative models, such as generative adversarial networks (GANs) and variational autoencoders (VAEs), can be used in inverse problems for data completion and sample augmentation, thereby improving inversion accuracy and generalization capability, and providing new methodological support for geophysical modeling.

Conclusion

This study systematically reviews the core role of mathematics in the modeling and analysis of geophysical phenomena, outlining the theoretical foundation for constructing mathematical structures, the modeling mechanisms of typical phenomena, and the adaptability of modern tools in data processing. The findings indicate that geophysical system modeling requires not only the accurate characterization of physical mechanisms but also relies on the expressive capability and computational efficiency of mathematical models under multiscale, coupled, and uncertain conditions. In scenarios such as atmospheric dynamics, seismic wave propagation, and gravity–magnetic inversion, partial differential equations, integral inverse problems, and numerical optimization methods provide solid support for phenomenon modeling. Facing the nonstationarity and complexity of observational data, multiscale analysis and high-order simulation methods enhance the analytical capability of models, while the introduction of machine learning techniques extends the expressive boundaries of models in structural identification, parameter inversion, and predictive estimation. Future research may further focus on optimizing the interpretability of mathematical models, deeply coupling intelligent algorithms with physical mechanisms, and ensuring modeling stability under high-dimensional big data conditions, thereby promoting the evolution of geophysical modeling toward intelligence, adaptability, and integration.

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