

# Design of Adaptive Mechanical Fixture and Optimization of Clamping Force for Complex Parts

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**Abstract:** *The continuous change of normal vectors and local curvature mutations of complex curved-surface parts cause instantaneous overdetermination or underdetermination of fixture degree-of-freedom constraints, and the force-deformation coupling combined with load time-varying further exacerbates the difficulty of clamping stability control. An adaptive mechanical fixture design and a clamping force optimization strategy are proposed: it establishes a method for characterizing degree-of-freedom fluctuations based on second-order differential geometric parameters and achieves dynamic matching of the configuration to the part morphology through adaptive contact element layout and topology-scale co-design; it constructs a nonlinear force-deformation mapping, solves the minimum norm solution of clamping force via convex quadratic programming, and introduces rolling horizon dynamic programming to adapt to load time-varying; and it designs a parametric scheme for variable stiffness flexible units, establishes a force-position stability criterion under excitation disturbances, and forms an adaptive closed-loop regulation mechanism for the coordination of stiffness and clamping force. This method treats stiffness as a control degree of freedom equal to clamping force, thus providing theoretical support for high-precision machining of complex parts.*

**Keywords:** *Adaptive Mechanical Fixture; Complex Parts; Clamping Force Optimization; Force-Deformation Coupling; Variable Stiffness Flexible Unit; Closed-Loop Stability Control*

## Introduction

Complex parts such as aero-engine blades and mold cores are widely used in mechanical processing, and their free-form surface regions and local concave structures impose stringent requirements on the adaptability of fixtures. The traditional six-point locating principle is based on the assumption of an ideal rigid body, and it cannot handle the degree-of-freedom fluctuation problem caused by the continuous change of the part surface normal vector, which often leads to local overconstraint or contact slippage. Meanwhile, the elastic deformation of thin-walled or cantilever structures under clamping force reconstructs the contact force distribution, forming a force-deformation nonlinear coupling, and the time-varying characteristics of cutting force, inertial force, and vibration during machining further disturb the positioning stability. Therefore, designing a fixture system that can adapt to part geometric features and dynamically optimize clamping force becomes a key technical requirement for improving the machining quality of complex parts. Existing research mostly focuses on static optimization of fixture layout or single-objective optimization of clamping force, and it rarely considers the fixture structural stiffness and clamping force as coordinated control variables. Moreover, it lacks a systematic criterion for force-position stability under excitation disturbances. This paper addresses the two core issues of geometric adaptability and clamping stability for complex parts. It conducts research from three aspects: fixture configuration generation, dynamic allocation of clamping force, and stiffness-force coordinated closed-loop regulation, thereby constructing a complete theoretical framework for adaptive fixture design and optimization.

## 1. Adaptive Fixture Configuration Generation under Geometric Feature Constraints of Complex Parts

### 1.1 A Method for Characterizing the Degree-of-Freedom Fluctuation of Complex Curved-Surface Parts

The free-form surface regions and local concave structures of complex curved-surface parts cause

deviations between the theoretical locating points and the actual contact states in the traditional six-point locating principle. This deviation originates from the continuous change of the part surface normal vector, which leads to instantaneous overdetermination or underdetermination of the degree-of-freedom constraints of the clamping system during the fixturing process. To quantify this characteristic, this paper adopts second-order differential geometric parameters of the surface, namely the principal curvature and Gaussian curvature, to numerically describe the local morphology of the part surface, and it establishes a degree-of-freedom contribution matrix for the tangent plane constraints at the contact points. The eigenvalue distribution of this matrix can reflect the potential fluctuation range of the part's translation and rotation amplitudes in each direction under no external force.

This method further expresses the degree-of-freedom fluctuation as the rate of change of the condition number of the constraint Jacobian matrix in the part coordinate system. For typical complex parts such as turbine blades or mold cores, the extreme curvature points and saddle point regions on the surface correspond to the critical transition states of the degree-of-freedom constraints. By constructing a discrete point set of surface sampling and calculating the singular value distribution of the constraint matrix at each point, this method obtains the statistical characteristic parameters of the overall degree-of-freedom fluctuation of the part, including the mean fluctuation amplitude and the local range. This characterization method provides quantitative input conditions for the subsequent layout optimization of the fixture contact elements, and it enables the adaptive fixture to actively adjust the spatial configuration of contact points according to the geometric fluctuation of the part surface<sup>[1]</sup>.

### ***1.2 Adaptive Layout Criteria of Contact Elements Based on Local Morphology of the Workpiece***

The layout of contact elements on the surface of a complex part directly determines whether the fixture system can reliably constrain the degrees of freedom of the part. The traditional principle of equal spacing or uniform distribution cannot meet the clamping requirements in regions with curvature mutations on the surface, which leads to local overconstraint or contact slippage. Therefore, this study proposes a contact element adaptation criterion based on the local morphology curvature radius, namely that the arrangement density of contact elements should be positively correlated with the second-order change rate of the part surface. In high-steepness regions with a small curvature radius, the distribution density of contact elements should be increased and the spacing between elements should be reduced, so as to suppress the possible release of rotational degrees of freedom of the part in that region.

This criterion further specifies that the geometric end shape of the contact element must match the local surface normal direction. For convex curved surface regions, a point-contact ball-type unit is adopted to avoid stress concentration caused by line contact; for concave curved surface or groove regions, a swingable roller-type contact head is used to achieve adaptive alignment of the contact normal force. By analyzing the normal vector and principal curvature direction of each node in the part surface mesh model, this method generates parametric constraint conditions for the spatial pose of the contact elements. This layout criterion enables the topological structure of the fixture contact system to dynamically adjust with the local morphology of the part, thereby maintaining a constant rank of the constraint Jacobian matrix globally.

### ***1.3 Topology and Scale Co-Design Strategy of the Reconfigurable Fixture Mechanism***

The reconfigurable fixture mechanism consists of multiple independently driven locating branches and clamping branches. The topological connection forms of the branches and the linkage scale parameters are coupled with each other, and they jointly affect the reachable workspace and stiffness distribution of the clamping system. The topology design focuses on determining the kinematic pair types of each branch and their serial or parallel connection relationships, and it preferentially adopts a four-bar linkage mechanism with planar redundant actuation characteristics or a six-degree-of-freedom serial manipulator configuration. The scale design targets the determined topology to optimize the lengths of each linkage and the joint offsets, so that the end contact elements of the fixture can cover all the surfaces to be clamped on the complex part<sup>[2]</sup>.

The topology and scale co-design is built upon a multi-objective optimization framework. The topological variables adopt discrete coding to represent the type of kinematic pairs and the connection sequence, and the scale variables adopt continuous variables to represent the geometric parameters of the linkages. This study constructs the objective functions to include the maximization of the global stiffness of the clamping system and the minimization of the driving torque fluctuation, and it sets the

constraint condition that the set of reachable positions of the contact elements must contain all candidate locating points on the part surface. A nested optimization strategy is adopted: the outer layer searches for the optimal topological structure through a genetic algorithm, and the inner layer solves a nonlinear programming problem for the scale parameters for each candidate topology. This co-design strategy avoids the workspace discontinuity problem caused by the separate design of topology and scale in traditional fixture design, and it enhances the geometric adaptability of the adaptive fixture to complex parts.

## **2. Dynamic Allocation Mechanism of Clamping Force and Multi-Objective Optimization Model**

### ***2.1 Nonlinear Mapping Relationship of Force-Deformation Coupling During the Clamping Process***

The elastic deformation of a complex part under clamping force changes the actual geometric morphology of the contact area, which in turn leads to a redistribution of the contact force. This force-deformation coupling effect cannot be described by the linear superposition principle, because the nonlinear characteristics of the local stiffness of the part surface intertwine with the micro-slip behavior of the contact interface. The establishment of this mapping relationship requires the introduction of Hertz contact theory from contact mechanics and the finite element node force transfer matrix. This study expresses the function between the clamping force vector and the node displacement vector of the part surface as an implicit differential equation. The core of this equation lies in the fact that the contact stiffness matrix depends on the current deformation state, which forms a typical nonlinear algebraic system.

To solve this coupled mapping, this study adopts an incremental iterative format and discretizes the clamping process into several load steps. Within each load step, the stiffness matrix is calculated based on the current contact configuration, and the contact boundary conditions are updated after solving for the displacement increment. The convergence criterion is set such that the relative change rate of the Euclidean norm of the contact force vector between adjacent iteration steps falls below a given threshold. For the thin-walled or cantilever structures that are characteristic of complex parts, this nonlinear mapping shows that a small increment of local clamping force may induce a sudden change of displacement in adjacent regions, that is, a bifurcation point appears on the force-deformation curve. Accurately capturing this nonlinear behavior serves as the theoretical premise for setting the constraint conditions in the subsequent clamping force optimization model.

### ***2.2 Minimum Norm Solution of Clamping Force for Workpiece Positioning Stability***

The workpiece positioning stability requires that neither detachment nor slippage occurs at all contact points, and the corresponding mathematical constraints are that the normal contact force is positive and the tangential force satisfies the Coulomb friction cone condition. Under the above constraints, there exist infinitely many feasible solutions for the clamping force, and the minimum norm solution specifically refers to the unique solution that minimizes the two-norm of the clamping force vector. This problem can be formulated as a convex quadratic programming model: the objective function minimizes the sum of squares of the clamping forces, the linear constraints come from the force balance equation and the moment balance equation, and the inequality constraints come from the tangential force limiting conditions after linearization of the friction cone. Since the number of workpiece locating points is usually greater than the required degrees of freedom of constraint, this optimization problem is redundant, and the minimum norm solution corresponds to the clamping scheme with the lowest energy consumption<sup>[3]</sup>.

To solve this quadratic programming problem, the dual method or the active set algorithm can be adopted. For complex parts, the minimum norm solution of clamping force often distributes the clamping force preferentially to the contact regions with higher stiffness, because a small force applied in a high-stiffness region can generate a sufficient constraint moment. However, this characteristic may cause some contact points to receive a near-zero clamping force, which makes them prone to detachment under dynamic loads. To balance this tendency, a regularization term needs to be added to the objective function, that is, a weighted norm replaces the standard two-norm, and the weight coefficient is inversely proportional to the local stiffness of the contact point. The modified minimum norm solution not only maintains the minimum clamping energy characteristic but also ensures a reasonable margin in the distribution of clamping forces at each contact point.

### ***2.3 Dynamic Programming Model of Clamping Force Considering Load Time-Varying Characteristics***

During the machining process, the cutting force, inertial force, and vibration load all exhibit time-varying characteristics, and the static clamping force distribution cannot adapt to the transient fluctuations of the loads. The dynamic programming model divides the entire machining period into several time intervals. Within each interval, the load is regarded as quasi-static, and the clamping force is allowed to be adjusted between different intervals. The state variables are defined as the current workpiece pose deviation and its derivative, and the control variable is the clamping force increment applied during that interval. The state transition equation is obtained by discretizing the dynamic equation of the clamping system, which includes the nonlinear term of force-deformation coupling. The objective function is set as the weighted sum of the accumulated square value of the workpiece positioning error over the entire time domain and the energy consumption for clamping force adjustment<sup>[4]</sup>.

The solution of the dynamic programming problem must address the challenges of a high-dimensional state space and nonlinear constraints. This study adopts a rolling horizon optimization strategy, which optimizes the clamping force sequence for only a limited number of future time intervals at each decision moment. The adjustment rate of the clamping force is limited to a finite value to avoid the saturation nonlinearity of the hydraulic valve or the motor driver in the servo system. The clamping force trajectory output by the dynamic programming exhibits a composite structure of feedforward plus feedback: the feedforward component adjusts the clamping force in advance according to the estimated load variation trend, and the feedback component provides real-time correction based on the actual pose deviation. This dynamic programming model enables the adaptive fixture to actively regulate the clamping force during the machining process, thereby compensating for the disturbance of load time-varying to the positioning stability.

## **3. Adaptive Fixture Stiffness Matching and Closed-Loop Stability Control**

### ***3.1 Parametric Configuration Design Method of Variable Stiffness Flexible Units***

The variable stiffness flexible unit is the core actuating element for achieving adaptive fixture clamping, and its stiffness needs to be continuously adjustable over a wide range. The parametric configuration design expresses the geometric shape and material distribution of the flexible unit as a set of independent design variables, including the thickness, length, and notch curvature of the flexure hinge, as well as the pre-compression amount of the stacked piezoelectric stack actuator. By changing the ratio of the equivalent bending stiffness to the axial compression stiffness of the flexible unit, this method obtains a quantitative expression of the stiffness adjustment range. The parametric model is established on the basis of Euler-Bernoulli beam theory, and it introduces a follower boundary condition to describe the force transmission path between the driving element and the flexible structure. Furthermore, this method considers the geometric nonlinear effect of the flexible unit under large deformation conditions, and it captures the drift characteristic of stiffness varying with load by using an incremental finite element format, thereby defining the boundary curve between the linear stiffness region and the nonlinear hardening region in the parameter space.

A scheme combining geometric parameterization and material parameterization is adopted. The geometric parameterization controls the topological configuration of the flexible unit, such as the profile curves of a triangular, circular-arc, or leaf-type flexure hinge; the material parameterization is realized through an embedded thin layer of shape memory alloy, and the equivalent elastic modulus of the unit can be adjusted by changing the martensite volume fraction of the alloy layer. The goal of the parametric design is to make the stiffness adjustment ratio of the flexible unit exceed one order of magnitude while keeping the contact point position offset during adjustment within the part positioning tolerance zone. A response surface method is used to establish a mapping surrogate model between the design variables and the stiffness output, which provides a fast calculation channel for subsequent stiffness matching. To improve the prediction accuracy of the surrogate model, a Kriging interpolation method is adopted to fit the residual term, and the maximum prediction variance is used as the criterion for pointwise sampling region refinement, so that the stiffness prediction error of the variable stiffness unit at key operating points is controlled within an engineering acceptable range<sup>[5]</sup>.

### ***3.2 Force-Position Stability Criterion of the Clamping System under Excitation Disturbances***

The dynamic cutting force between the tool and the workpiece during the machining process excites the fixture structure to produce a vibration response, and this response may cause micro-slip or instantaneous detachment at the contact points. The force-position stability criterion is used to judge whether a given clamping system can maintain contact integrity under specific disturbance frequency and amplitude conditions. Based on Lyapunov's second method, this study represents the clamping system as an autonomous system with the workpiece pose deviation as the state variable, and the stability of its equilibrium point is determined by the positive definiteness of the generalized stiffness matrix and the generalized damping matrix of the system. The excitation disturbance is regarded as a bounded input, and the input-output stability of the system is evaluated through the L2 gain. For a hydraulic fixture with time-delay effects, the Krasovskii functional is also introduced to deal with the stability boundary shrinkage phenomenon of the delay differential equation.

The specific criterion takes the form that the ratio of the tangential force to the normal force at the contact point must be less than the friction coefficient multiplied by a safety margin factor, and the peak value of the power spectral density of the contact force at the disturbance frequency does not exceed a certain percentage of the static normal force. For a multi-contact-point system, all contact points must satisfy the above conditions simultaneously. This criterion further derives an expression for the critical disturbance frequency, namely that when the excitation frequency approaches a certain natural frequency of the clamping system, the stability boundary shrinks sharply. Embedding this stability criterion into the fixture control system enables online stability monitoring under disturbance conditions. To improve the engineering practicality of the criterion, the modal participation factor is introduced to rank the modes of the system, and only the main modal frequency bands sensitive to contact force disturbance are monitored, thereby reducing the online computational burden and improving real-time performance<sup>[6]</sup>.

### ***3.3 Adaptive Closed-Loop Regulation Mechanism for Coordination of Fixture Structural Stiffness and Clamping Force***

Structural stiffness and clamping force exhibit a coupling effect: increasing the structural stiffness can reduce the demand for a high clamping force, but excessively high stiffness amplifies impact loads; increasing the clamping force can suppress contact slippage, but it aggravates workpiece deformation and increases energy consumption. The coordinated regulation mechanism requires dynamically matching the proportional relationship between the two under the premise of ensuring positioning stability. The closed-loop regulation mechanism consists of a sensing feedback link, a decision-making and planning link, and an actuation link. The sensing link monitors the contact force vector and the workpiece surface displacement field in real time. The decision-making link solves for the optimal stiffness-force combination based on the current load state. The actuation link simultaneously adjusts the stiffness setpoint of the variable stiffness unit and the output of the clamping force actuator. The innovation of this mechanism lies in treating stiffness as a control degree of freedom equal to clamping force, rather than as a preset fixed parameter.

The core of the decision-making link is a two-layer optimization structure. The upper layer determines the ideal ratio between stiffness and clamping force according to the type of machining stage and the current load characteristics. The lower layer optimizes the stiffness setpoint and the force amplitude respectively under the condition that this ratio is fixed. A model predictive control framework is adopted to transform the coordinated regulation problem into a rolling optimization within a finite time horizon. The predictive model includes the nonlinear dynamic equation of the clamping system and the hysteresis characteristic of the variable stiffness unit. The feedback correction link uses the deviation between the actual measured value and the predicted value to correct the control variable at the next moment. This closed-loop mechanism enables the fixture to adaptively respond to changes in machining conditions and to maintain clamping reliability while avoiding excessive workpiece deformation. An event-triggered control strategy is further introduced, which activates the optimization calculation only when the clamping force deviation exceeds a preset threshold, thereby significantly reducing the energy consumption and processor occupancy of the control system during the steady-state machining phase.

## Conclusion

This paper systematically expounds the design method of the adaptive mechanical fixture and the clamping force optimization strategy for the clamping difficulties in the machining of complex parts. At the configuration level, this paper proposes a method for characterizing the degree-of-freedom fluctuation of complex curved surfaces and a layout criterion for contact elements based on local morphology, and it achieves adaptive matching of the reconfigurable fixture mechanism to the part geometric features through topology and scale co-design. At the mechanical level, this paper establishes a nonlinear mapping relationship of force-deformation, presents the minimum norm solution of clamping force for positioning stability, and realizes active compensation for load time-varying characteristics by using a rolling horizon dynamic programming model. At the control level, this paper designs a parametric configuration scheme for the variable stiffness flexible unit, derives a force-position stability criterion under excitation disturbances, and further proposes an adaptive closed-loop regulation mechanism for the coordination of stiffness and clamping force.

The above methods break through the limitations of preset stiffness and static clamping force distribution in traditional fixture design, and they transform the fixture system into an intelligent actuation unit that can perceive the part morphology and load state and dynamically adjust its own mechanical behavior. Future research directions can include the following three aspects: extending the proposed theoretical model to multi-station assembly line fixture systems and studying the transfer and coordination mechanisms of clamping states between different stations; introducing a model-free adaptive control method based on deep reinforcement learning to address the online clamping force decision-making problem for complex parts with incompletely known geometric parameters or under strong uncertainties in the machining environment; and exploring new materials and driving methods for variable stiffness flexible units, such as dielectric elastomers or magnetorheological materials, to increase the stiffness adjustment range and response speed, and to further reduce the energy consumption and volume of the fixture system.

## Fund Projects

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