Physical Modeling of the Impact of Process Corner (PVT) Variations on Mismatch in Analog Integrated Circuits in Deep-Nanometer Technologies

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Abstract: *Mismatch at deep-nanometer process nodes arises from various sources including random doping, line edge roughness, metal-gate work function fluctuation, oxide thickness fluctuation, strain fields, and interface trap states. The supply voltage and temperature fluctuations control the mismatch sensitive parameters via bias dependence and temperature dependence which leads to non-Gaussian, non-stationary and anisotropic correlation in the statistics of threshold voltage, current factor, mobility, subthreshold slope and parasitic parameters. Square-root law and empirical statistical models do not cover their physical coupling mechanisms. We propose a physical modeling framework on mismatch with process corner variations. Solving Poisson equations and quantum-corrected carrier transport equations we build stochastic source terms; using sensitivity tensors and Jacobian operators, pushing forward the PVT perturbation field into covariance kernels of device parameters gives us both continuous and discrete formulation. A mismatch sensitivity tensor and a process corner perturbation propagation operator characterizes the coupling between nonlinear transconductance and parasitic parameter mismatch, providing statistical closure and error bounds for functional modules. This model offers a structure for the statistically describing mismatch in deep-nanometer analog integrated circuits in a physically interpretable way.*

Keywords: *deep-nanometer process; analog integrated circuits; process corner variations; mismatch; physical modeling; covariance kernel*

Introduction

Various sources process disturbances (random doping, line edge roughness, metal-gate work function fluctuation, oxide thickness fluctuation, strain fields, and interface trap states), supply voltage and temperature fluctuations impact the mismatch behavior in analog integrated circuits by affecting the channel electric field, depletion region, carrier transport, and parasitic network which leads to the statistical characteristics of threshold voltage, current factor, mobility, subthreshold slope, and parasitic parameters have bias dependence, temperature dependence, non-Gaussian characteristics, and non-stationary characteristics. The traditional mismatch models based on empirical fitting with an independent and identically distribution assumption are unable to capture the cross coupling between PVT fluctuations and process corner disturbances, spatial correlation length, and physical boundary constraints; they do not naturally link device physics and circuit functional statistics. We need a physical modeling of the effect of PVT fluctuations on mismatch under deep-nanometer processes for statistical design, robustness assessment, and process co-optimization of analog integrated circuits as a physically-motivated, analytically tractable, discretizable, and verifiable mapping chain from device transport equations to circuit-level responses.

1. Physical Coupling Mechanisms Between PVT Fluctuations and Mismatch Under Deep-Nanometer Processes

1.1 Multi-Source Statistical Characterization of Process Corner Disturbances in Deep-Nanometer Devices

Process corners in deep nanometer devices are: random doping fluctuations, line edge roughness,

metal gate work function fluctuations, oxide thickness fluctuations, strain field fluctuations and interface trap state density fluctuations. They have multiscale distributions (wafer level, chip level, layout level), global process shifts, local random parts and non-Gaussian tails. Joint probability density for a process corner parameter vector accounts for inter-source correlation, bias dependence and temperature dependence. Random doping distorts the channel potential barrier and depletion region charge by discrete position of impurities; line edge roughness influences the threshold voltage and current factor due to an effective channel length and width; metal-gate work function fluctuation changes the flat-band voltage and subthreshold slope via the spatial distribution of the work function[1].

Carrier mobility and band structure affected by strain fluctuations; Interface trap states induce low frequency noise and threshold drift via trapping/release. Quantum confinement, source-drain tunneling, velocity saturation and ballistic transport at deep nanometers scale differs than traditional square-root law mismatch model: Process corner disturbances are mapped with continuous random fields through Poisson equations, quantum-corrected drift-diffusion equations or non-equilibrium Green's functions. Correlation length, power spectral density, covariance matrix and higher order cumulants statistically characterize conversion of discrete microscopic disturbances in a random parameter set (of threshold voltage, current factor, mobility, subthreshold slope, and output conductance) that serve as input random sources for PVT modulation.

1.2 Mechanisms by Which Voltage and Temperature Fluctuations Modulate Mismatch-Sensitive Parameters

Statistical characteristics of mismatch sensitive parameters are affected by bias dependence and temperature dependence of supply voltage and temperature fluctuations. Fluctuation of supply voltage impacts the channel electric field, depletion region width, carrier distribution, and parasitic capacitance so that the sensitivities of threshold voltage mismatch, current factor mismatch, transconductance mismatch, and parasitic parameter mismatch have a bias dependent variation. Impact of random doping fluctuations and interface state fluctuations on subthreshold current becomes larger at lower voltages, drain induced barrier lowering and gate-induced drain leakage introduce additional random source, and the sensitivity of subthreshold slope mismatch with respect to interface state density has significant increase at this stage. Velocity saturation, channel length modulation, and hot-carrier effects change the weights of all components of the mismatch sensitivity tensor at higher voltages and the correlation between transconductance mismatch and output conductance mismatch increases when the drain voltage increases.

Variations of temperature impact lattice scattering, phonon occupancy, thermal voltage, intrinsic carrier concentration, contact resistance causing carrier mobility, temperature coefficient of threshold voltage and subthreshold slope to drift with temperature. Increasing temperature enhances contribution from interface states and trap-assisted tunneling while decreasing temperature enhances proportion of random doping and quantum confinement effects in mismatch at deep nanometer nodes. The modulation of mismatch due to PVT fluctuation is captured by representing sensitivity tensor for mismatch-sensitive parameters w.r.t. voltage and temperature which modulate random field of process corner disturbances and reweight each source of mismatch. Cross coupling exists between voltage-temperature disturbances and process corner disturbances, bias point and temperature point alters the shape and correlation length of the mismatch covariance kernel, and cross-coupling terms considered through second order sensitivities (mixed partial derivatives) in the modulation model. Considering voltage and temperature as parameters of stochastic differential equations or hyperparameters of the covariance kernel enables establishing a continuous modulation model of mismatch sensitive parameters varying with PVT linking multi-source statistical characterization of process corner disturbances with circuit level mismatch response[2].

1.3 Spatial Correlation and Physical Boundary Conditions of Mismatch Random Fields

The mismatch appears as a random field in the layout space that correlates by the process disturbance correlation length, device spacing, layout topology, and physical boundaries. The correlation lengths and directions of random doping, line edge roughness, and metal-gate work function fluctuation lead to different anisotropic correlations between threshold voltage mismatch and current factor mismatch. The random doping correlation length depends on the discrete distribution of impurities in the channel and the annealing diffusion length while the line edge roughness correlation length has significant difference along the gate-width and gate-length direction; Whereas the metal-gate work function fluctuation correlation length matches the grain size distribution. Superposition of wafer

level gradients with chip-level systematic shift and local random part leads to the mismatch random field being non-stationary whose spatial structure need description via variogram, covariance function, or spectral density. Because of the influence of discrete impurities and quantum confinement effect, for deep-nanometer devices the random field deviate from a Gaussian distribution, which can be described through conditional random fields and hierarchical Bayesian descriptions describe the propagation of local disturbances inside device geometry under constraints.

Physical boundary conditions are: gate boundaries, source-drain boundaries, oxide interfaces, substrate contacts, interconnect parasitic boundaries which truncate/reflect random disturbances changing the shape of the mismatch covariance kernel at device edges and layout boundaries. Boundary conditions of Poisson equations and carrier transport equations restrict the range of microscopic disturbances in accessible state space defined by material interfaces and device geometry (a physically feasible domain of the mismatch random field); spatial correlation length and boundary conditions impose matching properties of mismatch between neighboring devices with geometric and physical constraints for covariance kernel mapping, circuit level mismatch propagation and statistical model closure; distortions due to truncation effect from boundary condition on the random field requires introduction of boundary correction terms when building up the covariance kernel to ensure statistical consistency[3].

2. Physical Modeling Framework for Mismatch Under Process Corner Variations

2.1 Construction of Mismatch Source Terms Based on Carrier Transport Equations

Starting with coupled Poisson equations and carrier continuity equations for deep-nanometer devices, we incorporate random doping, line edge roughness, metal-gate work function fluctuation, oxide thickness fluctuation, strain fields, and interface trap states into drift-diffusion equations by introducing random coefficients or random source terms: random doping influences the space charge density via an ionized impurity charge term; line edge roughness modifies the boundary geometry and scattering rate via effective channel length and width; work function fluctuation impacts the flat-band voltage via gate boundary conditions; trap states adds a random recombination rate via Shockley-Read-Hall recombination term and trapping kinetics. The deviation on classical drift-diffusion equations caused by quantum confinement, source-drain tunneling, and ballistic transport lead us to adopt quantum-corrected drift-diffusion equations, Boltzmann transport equations, or non-equilibrium Green's functions to project random disturbances onto random components of effective mass, mobility, barrier height, and density of states.

The transport equation mismatches arise from a sum of random coefficient and source fields whose statistics depend on the joint probability density and spatial correlation structure of multi-source disturbances. Source term construction uses random variable decomposition that breaks down process corner disturbances into deterministic process corner shift and zero-mean random fluctuation (first one change the operating point and sensitivity; second one introduce mismatches) and cross dependence with bias voltage and temperature via mobility models, saturation velocity models, thermal voltage, and temperature coefficients: we construct a set of random source terms for threshold voltage, current factor, subthreshold slope, transconductance, and output conductance giving physical interpretation for covariance kernel mapping[4].

2.2 Mapping Between PVT Disturbance Fields and Device Parameter Covariance Kernels

PVT disturbance field includes process corner random field, supply voltage fluctuation and temperature fluctuation with statistics such as mean, covariance function, correlation length, anisotropy and higher order cumulants. The device parameter covariance kernel captures the auto-covariance and cross-covariance between threshold voltage, current factor, mobility, subthreshold slope, parasitic capacitance and parasitic resistance. Mapping process uses sensitivity tensor (Jacobian) operator pushing the covariance function from the PVT disturbance field to device parameters space creating a bias- and temperature dependent covariance kernel. Voltage and temperature act as hyperparameter changing the elements in Jacobian matrix leading to variation on the amplitude, correlation length and directionality of covariance kernel depending on the operating point. Cross terms associated with process corner disturbances and voltage-temperature disturbances are captured by second-order sensitivities or mixed partial derivatives without introducing any statistical bias because of independence assumption.

Covariance kernel mapping handles non-Gaussian/non-stationary feature: random doping/discrete impurity induces non-Gaussian tail of deep-nanometer devices, line edge roughness/metal gate work function fluctuation have directional correlation, wafer level gradient/chip level systematic shift brings about non-stationary mean. Covariance kernel mapping leverages cumulant expansion, conditional covariance functions/hierarchical Bayesian pushforward to combine local random fields/global process corner shifts into device parameter covariance kernel. Kernel function truncates/reflects on layout boundary/device geometry constraint that forms covariance structure within physical feasible domain. Mapping connects PVT disturbance field with mismatch source term and provides target statistics for discretization scheme.

2.2 Mapping Between PVT Disturbance Fields and Device Parameter Covariance Kernels

PVT disturbance field: Process corner random field, supply voltage fluctuation, temperature fluctuation – mean, covariance function, correlation length, anisotropy, higher-order cumulants Device parameter covariance kernel: Auto-covariance and cross-covariance on threshold voltage, current factor, mobility, subthreshold slope, parasitic capacitance, parasitic resistance Sensitivity tensor (Jacobian): Maps covariance function of PVT disturbance field into device parameter space to obtain bias- and temperature-dependent covariance kernel Voltage and temperature act as hyperparameters which modify the entries in Jacobian matrix such that amplitude, correlation length and directionality of covariance kernel vary with operating point. Cross terms between process corner disturbances and voltage-temperature disturbances are accounted by using second order sensitivities (mixed partial derivatives); this avoids statistical bias arising out of independence assumption.

Covariance kernel mapping should handle non-Gaussian and non-stationary features: Non-Gaussian tails emerge from random doping and discrete impurities in deep-nanometer devices; Directional correlation structures arise because of line edge roughness and metal gate work function fluctuation; Non-stationary means come from wafer level gradients and chip-level systematic shifts. Cumulant expansion, conditional covariance functions or hierarchical Bayesian pushforward unify local random fields and global process corner shifts within the device parameter covariance kernel. Kernel function truncates or reflects on the layout boundary and device geometry constraint as a covariance structure in the physically feasible domain. Links the PVT disturbance field with mismatch source terms and deliver target statistics for the discretization scheme.

2.3 Continuous and Discrete Formulations of the Mismatch Model at Deep-Nanometer Scales

The mismatch sources, PVT disturbances can be described by stochastic partial differential equations, random field models; Poisson equations, quantum-corrected transport equations and boundary conditions are continuous operators including random doping/interface traps/work function fluctuations into its random coefficient fields; Statistical moments of mismatch sensitive parameters can be described by continuous covariance equations/Fokker-Planck equations where covariance kernel is evolving based on the evolution equations derived from transport operators. The continuous formulation preserves the quantum confinement, ballistic transport and tunneling effects for deep nanometer devices such that the mismatch model maintains physical consistency at geometric boundaries/material interfaces. Voltage and temperature fluctuation acts like parameter/operator changes (stochastic process), changing the coefficients/boundary condition of the continuous model forming a bias/temperature dependent statistical description.

Discrete formulation transforms the continuous stochastic model into computable algebraic form: Finite volume method preserves currents and fluxes; finite element method handles complex gate and source-drain geometries; spectral methods efficiently compute the covariance kernel in stochastic dimensions; stochastic collocation, sparse grids, and polynomial chaos expansion deal with high dimensional PVT disturbance space. Deep nanometer scale needs discrete grids for resolving discrete impurities, interface traps, and quantum potential barriers and discrete operators preserving positive definiteness, symmetry, and boundary constraints of the covariance kernel. The discrete errors are controlled via a posteriori error estimation and covariance convergence criteria such that device parameter statistics stay stable when mapped at circuit level. Provides a systematic approach from continuous physical models to its numerical implementation[5].

3. Physical Model Analysis and Circuit-Level Mapping of Mismatch Responses in Analog Integrated Circuits

3.1 Mismatch Sensitivity Tensors and Process Corner Disturbance Propagation Operators

Mismatch sensitivity tensor consists of partial derivative relationships from circuit functional parameters (differential gain, common-mode rejection ratio, input offset, bandwidth, phase margin, noise figure, settling time) with device mismatch source parameters (threshold voltage, current factor, mobility, subthreshold slope, parasitic capacitance, parasitic resistance). Sensitivity tensors change based on bias, temperature, and frequency at deep nanometers: they contain first-, second-order as well as mixed partial derivatives which need to be computed via solving small-signal equivalent circuits, using the adjoint network method, or automatic differentiation. PVT fluctuations are tensor hyperparameters changing the bias point and temperature point redistributing the weights for each mismatch source on circuit functional parameters.

Process corner disturbance propagation operator transforms device level random disturbance field to circuit level random response; composed by Jacobian matrix of circuit equations/state-space/transfer function matrices. It maps the device parameter covariance kernel into covariance matrix of circuit functional parameters, keeping correlation structure changes caused by PVT fluctuation. Static propagation is based on DC Jacobian matrix, dynamic propagation is based on state space model or frequency domain transfer functions. PVT fluctuations are used as parameters/operator/random input changing eigenvalues/singular values within the propagation operator altering variance/correlation length of mismatch responses. After separating process corner shift and random fluctuation, deterministic shifts change the operating point of the propagation operator, random fluctuations are amplified or suppressed through the operator gain. The combination of propagation operator and sensitivity tensor constitute the core of circuit level mismatch mapping: They give an operator basis for analyzing nonlinear transconductance and parasitic parameter coupling.

3.2 Coupled Analysis of Nonlinear Transconductance and Parasitic Parameter Mismatch

Transconductance mismatch caused by random threshold voltage variation, mobility, carrier velocity saturation, channel length modulation and quantum confinement. Nonlinear transconductance mismatch affects analog circuits via small-signal gain, linearity, harmonic distortion, and intermodulation distortion. Sensitivity of transconductance to bias voltage and temperature grows at deep nanometers; PVT fluctuation changes the second- and third-order derivative of transconductance so that the mismatch responses become biased dependent nonlinearities. Transconductance mismatch couples with output conductance mismatch and junction capacitance mismatch using an active device equivalent circuit as a joint statistical distribution for gain, bandwidth, and phase response. Coupled analysis requires high order transconductance terms and cross partial derivatives in the Jacobian matrix describing the nonlinear transformation of mismatch responses around the large-signal operating point.

Parasitic parameter mismatch: Random fluctuations on gate capacitance, junction capacitance, interconnect capacitance, contact resistance, via resistance. Parasitic parameter mismatch affects frequency response, group delay, settling behaviors, and noise coupling paths over RC network. Cross partial derivatives in Jacobian matrix reflect coupling effect between nonlinear transconductance and parasitic parameter mismatch -- transconductance mismatch alters node impedance and pole position; parasitic mismatch affects feedback paths and coupling coefficients. Interconnect parasitics and device parasitics have similar orders of magnitude in deep-nanometer processes requiring coupled analysis by integrating device physical models with interconnect electromagnetic models into an overall frequency domain and time domain consistent model describing mismatch propagation. Coupled analysis enables statistical closure of functional modules at component level.

3.3 Physical Model Closure and Error Bounds for Mismatch Statistics of Analog Functional Modules

The physical model closure for mismatch statistics of analog functional modules provides a self-consistency between device level covariance kernels, circuit-level propagation operators and functional parameter statistics. The analog functional modules include: differential pairs, current mirrors, operational transconductance amplifiers, bandgap references, sample-and-hold circuits, and oscillators. Physical model closure requires that pushing the device parameter covariance kernel by the propagation operator matches the statistical moments of circuit functional parameters with passivity

and positive definiteness constraints. Moment matching, cumulant truncation, or stochastic Galerkin projection maps the high dimensional PVT disturbance space onto a low dimensionality functional parameter space. Non-Gaussian tails and bias temperature dependence in deep nanometers requires closure to retain higher order cumulants or utilize conditional covariances to maintain the physical consistency of functional module statistics.

Error bounds are as follows: Model truncation error; Discretization error; Propagation operator linearization error; Covariance kernel estimation error. Truncation error arises from the order of cumulant expansion and the order of polynomial chaos, discretization error arises from device and interconnect meshes, linearization error arises from the neglected higher-order terms in the Jacobian matrix, and estimation error arises from finite samples and uncertainty in correlation length. Error bounds may be guaranteed through a posteriori error estimation, convergence analysis, and sensitivity ranking so that the mismatch statistics of functional modules are contained in an acceptable range at a certain confidence level. Physical model closure and error bounds provide a verifiable description for circuit-level mismatch responses, offering a physical basis on the statistical design and robustness analysis of analog integrated circuits.

Conclusion

This paper has presented our study on how process corner variation affects mismatch in analog integrated circuits at deep nanometers through developing physical couplings from multi-source statistical characterization and voltage-temperature modulation all the way to spatial random fields and boundary conditions, building mismatch source terms based on carrier transport equations, presenting mappings between PVT disturbance fields and device parameter covariance kernels as well as their continuous and discrete formulations, creating an architecture which maps device physics to circuit level mismatch response through mismatch sensitivity tensors, process corner disturbance propagation operators, coupled analysis of nonlinear transconductance and parasitic parameters, statistical closure and error bounds for functional modules. Future work could include 3D integration, heterojunction devices, and high frequency and low temperature operating condition; introduce multiphysics coupling, higher-order statistics of non-Gaussian random fields, model order reduction, and efficient stochastic algorithms, extend to a joint description of interconnect electromagnetics and device quantum transport, improve error bound and convergence theory enabling statistical modeling and design verification of more complex analog functional modules.

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