Reconstruction of Teaching Content for Linear Algebra in the Context of Artificial Intelligence: From Pure Computation to Matrix Thinking

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Abstract: *With the breakthrough of artificial intelligence technology, linear algebra's mathematical foundation, running through each aspect of model construction and semantic representation, has been greatly transformed. Most curricula are still based on the old tradition by emphasizing proficiency in symbolic computation without satisfying the requirement of data science for high dimensional geometric intuition and structural insight. Based upon that, we proposed a teaching reconstruction path from pure computation to matrix thinking; defined matrix thinking as a three level connotation (a data structure language, a spatial mapping tool, and an information processing paradigm); reshaped the three major modules of systems of linear equations, matrix factorization, and the eigenvalue problem respectively; promoted the return of linear algebra from being an algorithmic tool into a thinking framework; laid a mathematical cognition foundation for cultivating artificial intelligence talents.*

Keywords: *Artificial intelligence; Linear Algebra; reconstruction of teaching content; matrix thinking; data dimensionality reduction; eigenvalue problem*

Introduction

Linear algebra is indispensable in modern mathematics and one of the cornerstone of natural sciences and engineering technology. With the rapid development of artificial intelligence technology, there are new demands for the linear algebra knowledge system: Word embedding in deep learning models is distribution points in a low dimensional space; Convolution operation is equal to linear mapping with weight sharing; Attention mechanism aggregates information by linear transformation... All of them show that linear algebra is not just a calculation tool to implement algorithms, but the grammar to build the model's representational capacity. But today's curriculum still uses determinant computation and vector space axioms as the core module and emphasizes rigorous derivation of algebraic structure; learners' cognition about matrices is limited to two-dimensional numeric array and operation rule, lacking the habit of treating matrices as data container and linear mappings as channel of information flow. Such cognition limit greatly limits their ability to understand and innovate artificial intelligence algorithms. It is very significant to reconstruct the teaching content from a pure computing perspective to a matrix-thinking perspective under such background. In this paper we will start from the innovative demand that artificial intelligence puts forward for the linear algebra knowledge system, expounding the theoretical connotation and hierarchy of matrix thinking, and then give a reconstruction plan of teaching content modules oriented towards cultivating thinking.

1. Innovative Demands of Artificial Intelligence on the Linear Algebra Knowledge System

1.1 Evolution of the Linear Algebra Core in Artificial Intelligence Models

Core breakthroughs for current artificial intelligence technology emerge from massive development of deep neural network architecture that have fundamentally transformed the role of linear algebra as its mathematical foundation—representation, transformation and learning of high dimensional data are at the heart of how deep learning works; word embeddings in embedding layers are actually point distribution within a low dimension continuous space; convolution operations are basically linear

mapping with weight sharing; attention mechanisms select information by executing linear transformations on queries, keys and values··· all these parts of models indicate an obvious fact—the layer-by-layer stacking and nonlinear activations of neural networks are essentially doing a series of complex linear transformations and nonlinear mappings in a high-dimensional vector space. Linear algebra is not just a way to implement algorithms anymore—it's become the grammar to build model representational capacity — from tensor computation framework to geometric interpretation of optimization algorithm, its concept permeates throughout the whole lifecycle of artificial intelligence models.

Besides, the evolution path of linear algebra core changes from static structure to dynamic semantics: traditionally, we consider matrix and vector are static mathematical objects storing and calculating numbers but in modern generative models (e.g., generative adversarial network or variational autoencoder), interpolation and arithmetic operation over latent space vectors is a continuous change of semantic attributes while the low rank property of matrices explain the generalization ability and memorization behavior of model. We see an evolutionary trend from linear algebra as background computing support to foreground concept which become the linguistic basis for understanding the behavioral mechanism of artificial intelligence models.

1.2 Adaptive Limitations of the Traditional Teaching Paradigm to the Characteristics of Data Science

The existing Linear Algebra curriculum system mostly retains the classic scheme from the mid-twentieth century that focuses on calculating determinants, matrix operation and vector space axioms as its core modules; emphasizes strict derivation for algebraic structures and cultivates skills at symbolic computation. It fits well for traditional engineering and physics disciplines but with new requirements put upon the knowledge of linear algebra by the fields of data science and artificial intelligence, there are many shortcomings about adaptability. The core features of data science involve high dimensionality, massive scale and potential low-dimensional structure of data which requires learners have the capability to understand data processing workflow based on the geometric intuition of high dimensional spaces and treat the distribution, similarity and transformation of datasets as the movement of point sets in vector spaces instead of doing mechanical numerical calculations.

The emphasis on basis based coordinates and explicit calculation of transformation matrices is a key component of traditional teaching. Modern data tasks involve kernel mappings, random projections, or latent space transformations which cannot be explicitly represented. All core algorithms like data dimensionality reduction, feature extraction, and matrix factorization for recommender systems are approximations and interpretations of the low rank structure of matrices but traditional courses either address this topic only theoretically extended or fail to incorporate it organically into basic linear algebra concepts. The learners' understanding of matrices remains restricted to two dimensional numerical array with operations rules and they lack the thinking habit to see matrices as data container and linear mapping as channel of information flow. Such cognitive limitations restrict learners' understanding and innovation of artificial intelligence algorithms[1].

1.3 Ontological Return from a Tool for Algorithm Implementation to a Thinking Framework

To meet new demands on linear algebra knowledge system put forward by artificial intelligence technology, we need to rebuild the teaching content into the deeper dimension of cognition about the essence of the discipline, bringing back linear algebra from being instrumentality of implementing algorithms to becoming an ontology of a thinking framework. Thinking framework is not particular techniques nor memorized theorems, rather it is a way of seeing things, representing information and building models. It gives us a description language which mainly consists of vectors, matrices, spaces and transformations: Datasets in data science can be easily arranged as matrices; Relationships between features can be seen as vector inner product; Iterative process of algorithms can be considered as trajectory evolution of vectors under some dynamics. To internalize linear algebra as a thinking framework means that when facing new problems, learners will instinctively use spatial intuition for concept formation, treat data objects as points in space and data processing as transformation of point sets.

The change should shift our teaching contents from formal attention to computational process to mean-focused attention on concept's essence: Matrix is not only a rectangle storage for numbers anymore but also a carrier of data relationship and executor of transformation effect; Vector space is not

only an abstract algebraic structure anymore but the locus of data existence and the source of geometric intuition; Eigenvalue problem is no longer about solving characteristic polynomial but become the main components and energy distribution of data; Matrix factorization is no longer about simplifying computation but reveal the mathematical essence of information compression and feature extraction ... When we have shifted perspective in ontological level, linear algebra will really be integrated into the core knowledge fabric of artificial intelligence talent cultivation[2].

2. Theoretical Connotation and Hierarchical Architecture of Matrix Thinking

2.1 The Matrix Representation View as a Data Structure Language

Matrix Thinking on First Level is expressing data with matrices. Data sets, images, texts or graphs in the field of artificial intelligence and data science have capability to organize and express by matrices: The rows of a sample-feature matrix are the observed individuals but the columns denote the attribute dimension; we can think of a grayscale image as a pixel intensity matrix; Every row of a word embedding matrix carries the semantic vector for a word. It not only just orderings -- it gives data an algebraic structure. Row space and column space of a matrix shows linear association pattern between variables, the rank of a matrix represents the effective degrees of freedom of data, the inner product operation denotes the similarity measure among samples. We use matrices to describe and reason about the relationship of these data objects using the language of linear algebra. They make a bridge connecting raw data and mathematical theory.

Enhanced matrix representation view with sensitivity and grasp on data's characteristic. Many high dimensional data has low dimension manifold structures or sparse characteristics that are representable by low rank/sparsity/specific non-negativity constraint using matrix language such as: The user-item rating matrix in recommender systems naturally possesses low rank; The co-occurrence matrix in natural language processing tend towards sparse distribution; Images have sparse representations under some transform bases. Characteristics of data structure and matrices' algebraic properties provide learners understand about the essence of data from matrix analysis, giving cognitive basis for future modeling and algorithm design. Matrix is not just a container of data but also a mirror of the internal structure of data.

2.2 The Linear Transformation View as a Spatial Mapping Tool

At our second level we consider matrices as executors for linear transformations -- spatially define a linear mapping between two vector spaces (column space represents the image space), null space reflect the information loss. Convolution kernel within a convolutional neural network is just a linear transformation on a local receptive field, weight matrix of a fully connected layer recombines and project feature spaces, state transition matrix of a recurrent neural network control the evolution path of hidden states. If we treat matrices as spatial mappings, then these learners who think about algorithms would have geometric picture of transformation relationship among input space, feature space, and output space thus transcend their attention only at the numeric value of the parameter matrices itself.

Besides, there are in-depth understanding on transformation properties under the linear transformation view: Eigenvalues/eigenvectors is the invariant direction and scaling factor for a transformation (principal directions and variance explained by principal component analysis), Singular value decomposition is the gain coefficients of a transformation along different orthogonal directions (energy distribution of data over various modes) etc., learners will inevitably ask what's the geometric effect of their given weight matrix in a neural network or the distribution characteristic of its spectral structure and spatial form after its action if they face them; these questions show a jump in the level of linear algebra cognition from symbol operation to space thinking. Linear transformations go beyond being just an abstract mathematical definition becoming conceptual tool to understand the mechanism of information flow and feature extraction[3].

2.3 The Low-Rank Approximation and Sparsification View as Information Processing

At the third level of matrix thinking we consider matrices as instruments for information processing with their essence lying on understanding two basic processing paradigms – low-rank approximation and sparsification. Low rank approximation theory claims that most large-scale matrices carrying information have an essentially low dimension structure which can be well approximated by low-rank

matrices without loss of major information. Principal component analysis is a low-rank approximation of the covariance matrix; latent semantic analysis is a truncated singular value decomposition of the term-document matrix; the compression and acceleration of neural networks depends upon low-rank decomposition of weight matrices. To think about matrices as being approximable means that learners have cognitively made the shift from exact calculations to approximate representations, tolerating some controlled errors but gaining in computational efficiency and generalization ability.

The focus of Sparsification is concentration of information in proper representation: The Compressed sensing theory proves we can reconstruct a sparse signal using far less measurement compared to the requirement by Nyquist sampling theorem; Sparse coding learns features from sparse representation of data; Attention mechanism improves computation efficiency via sparsifying similarity matrices. In this context, matrix thinking will help learners to concentrate on the choice of representation and condense information (matrices are not only carrying data anymore but also information containers which could be transformed, compressed or sparsified); low rank and sparsity -- two complementary concepts for information processing, form the methodology foundation of modern artificial intelligence algorithm design; internalize them as organic elements of matrix thinking is one of the core tasks of teaching content reconstruction.

3. Reconstruction of Teaching Content Modules Oriented Toward Cultivating Thinking

3.1 Integration of Logical Orientation and Geometric Intuition in Solving Systems of Linear Equations

The traditional teaching for solving systems of linear equations emphasizes step by step training in eliminating methods and determining the existence of solutions: Students usually view the solution process as mechanically memorize the procedure of algorithmic operations. Reconstruction path towards matrix thinking includes explaining solution problems through the geometric framework of vector spaces and linear transformations where Gaussian elimination is inherently related with key concepts like the rank of a matrix, column space and null space. Solving a system of linear equations is actually finding the coefficient of a linear representation of a given vector using the column vectors of a matrix while the number and structure of solutions depends upon if the target vector lies inside the column space and the dimension of the null space. To raise the solution process from symbolic calculation to discuss the spatial relationship helps learners to build up a cognitive channel between algorithmic operations and geometric intuition and understand the spatial meaning behind every elimination step instead of just memorize operation procedures.

Incorporating geometric intuition can also be seen from the spatial understanding of solution results: unique solution means the bijective property of the transformation; infinitely many solutions mean there exists a null space in the transformation; no-solution will lead to solve it approximately in the least squares sense. Learners can understand the mathematical essence of data fitting and information redundancy between overdetermined systems and underdetermined systems. By combining the solution process with the dimension theorem of vector spaces and the rank-nullity theorem, the traditional introductory content of systems of linear equations take over the role of spatial thinking training and become the logical starting point for building up matrix thinking[4].

3.2 Algorithmic Perspective and Mechanistic Depth of Matrix Factorization Theory

The Matrix factorization plays as a bridge linking up not only linear algebra theory but also artificial intelligence algorithms; raising its reconstruction of teaching content turns it from just an algebraic identity manipulation to become a concept of understanding algorithm mechanisms. LU decomposition shows how Gaussian elimination process can be expressed by the language of matrices, QR decomposition embodies the core function of orthogonalization methods on numerical stability: they are all expressions of matrices based upon some algorithmic requirements. For artificial intelligence, relationships among nonnegative matrix factorization and topic extraction in topic models, equivalence between singular value decomposition and principal component analysis, association between spectral decomposition and convolution operations in graph neural networks are the mathematical basis to explain algorithm design ideas. By combining decomposition theory together with algorithm motivation we will let our learners ask if there exists this kind of matrix factorization form given a specific model so that they grasp the design intention and applicable scope of the algorithm.

In order to get deeper insights through decomposition theory, we must go further than formal proof on existence/uniqueness of decomposition into semantic interpretation of decomposition results. Left singular vectors/right singular vectors in singular value decomposition represent respectively the principal modes of samples and the principal modes of features; the decay rate of the singular value spectrum captures the intrinsic complexity of the data. Low-rank factorization form of a matrix represents the low-dimensional representation assumption of data which along with nonnegativity constraint gives decomposition result a semantic basis for interpretability. Mechanically analyze decomposition results so that learners can know that matrix factorization is not just simplifying computation but it also provides a conceptual lens to unveil data structure and information distribution: matrix thinking has acquired operable analytical tools.

3.3 Orientation of the Eigenvalue Problem Toward Data Dimensionality Reduction and Pattern Extraction

Traditional teaching focuses on solving the characteristic polynomial and judging the condition for diagonalization. Learner's cognition still stays at the level of algebraic calculation which makes it hard for learners to touch upon the geometric essence and information connotation of the eigenvalue concept. When reconstructing the teaching content, we should put the eigenvalue problem into the mathematical formalism of data dimensionality reduction and pattern extraction: eigenvectors are the principal direction of data distribution whereas eigenvalues denote the energy (variance) contribution along these directions. The principal component analysis is essentially the eigendecomposition of the covariance matrix, spectral clustering depends on the eigenstructure of the graph Laplacian matrix, the core of the PageRank algorithm lies in its principal eigenvector of the transition matrix. These classic algorithms in the field of artificial intelligence take the eigenvalue problem as their mathematical core. Transfer the eigenvalue concept from an abstract algebraic definition to the geometric interpretation of data distribution is a crucial link in constructing matrix thinking.

Developing sensitivity toward spectral structure: Eigendecomposition of real symmetric matrices leads to energy decoupling via orthogonal transformation, asymmetric matrices lead to complex eigenvalues with rotational dynamics; Spectral distribution of large-scale matrices dictate the convergence behavior of iterative algorithms. Spectral radius of weight matrices closely related to stability of gradient propagation in neural network theory or design intention of residual connections is controlling eigenvalue shift. Reexamine the eigenvalue problem helps learners develop a view on data covariance structure to algorithm's dynamic behavior; eigenvalues are not only roots of the characteristic equation anymore, but also core parameters to understand information distribution and system evolution.

Conclusion

With respect to reconstructing the teaching content of Linear Algebra with regard to artificial intelligence, we discuss it systematically in terms of innovative demands on the knowledge system, theoretical connotation of matrix thinking, and reshaping of teaching content modules. Our research reveals that the linear algebra core in artificial intelligence models has changed from static numerical computing to dynamic semantic representation, and the traditional teaching paradigm restricts adaptive cultivation of high dimensional data characteristics and spatial thinking, so there is an inevitable choice to promote the ontological return of linear algebra from being a tool for implementing algorithms to a thinking framework. We define matrix thinking as having three progressive levels (data structure language, spatial mapping tool, and information processing paradigm) corresponding to the representational function, transformation function, and approximation function of matrices, respectively.

Therefore, in order to solve a system of linear equations we need both logical orientation and geometric intuition; while in studying matrix factorization theory we need both algorithmic perspective and mechanistic depth; in studying the eigenvalue problem we need an orientation towards data dimensionality reduction and pattern extraction which turns abstract algebraic concepts into conceptual tools for understanding the behavioral mechanisms of artificial intelligence models. We could explore the cognitive laws and evaluation mechanism of cultivating matrix thinking or differentiate the teaching contents according to different artificial intelligence specialty directions such that our Linear Algebra course really becomes the core field for generating the mathematical literacy of artificial intelligence talents.

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