

# Study of Spin-Wave Mode Coupling and Band Structure in Magnetostatic Crystal

Xiaoying Hu\*

*School of Physics and Electronic Engineering, Jining Normal University, Ulanqab, 012000, China*

*\*Corresponding author: 15904849276@163.com*

**Abstract:** Magnetostatic crystals are magnetic materials with a periodic structure, and the coupling of their spin-wave modes and band structure has a significant impact on their dynamic behavior and applications. This paper investigates the spin-wave mode coupling mechanism, the evolution of the band structure, and the regulation methods within magnetostatic crystals. It explores their potential applications in fields such as spintronics and quantum computing. The study shows that the coupling of spin waves depends not only on exchange interactions but is also influenced by magnetic anisotropy, nonlinear effects, and external fields. The coupling of spin-wave modes can lead to the reconstruction of the band structure, affecting the propagation characteristics and stability of spin waves. Methods for regulating the band structure, such as external magnetic field adjustments, crystal geometric design, and nonlinear coupling, provide new pathways for optimizing spin-wave propagation characteristics. These studies offer theoretical support and a technical foundation for fields such as quantum information processing, low-power spin-wave communication, and spin current control.

**Keywords:** Magnetostatic crystal; Spin-wave mode coupling; Band structure; Nonlinear effects; Spintronics; Quantum computing

## Introduction

Magnetostatic crystals, as a new type of magnetic material, exhibit broad application prospects in spintronics and quantum information fields due to their unique spin-wave modes and band structures. Studying the spin-wave mode coupling and band structure in magnetostatic crystals not only helps deepen the understanding of spin-wave propagation mechanisms in magnetic materials, but also provides new theoretical foundations for developing low-power, high-performance spin-wave communication and logic computation technologies. With the rapid development of quantum computing and spintronics, the regulation of spin-wave modes in magnetostatic crystals has become a key technology for achieving efficient information transmission and processing. Particularly in frontier areas such as quantum bit storage, quantum logic operations, and spin-wave computing, magnetostatic crystals, through precise control of spin-wave propagation characteristics, can significantly enhance the stability and transmission efficiency of systems. Therefore, in-depth research on the spin-wave coupling effects and their impact on the band structure holds significant academic value and practical implications.

## 1. Basic Theory and Research Background of Magnetostatic Crystals

### 1.1 Physical Properties of Magnetostatic Crystals

Magnetostatic crystals are magnetic materials with a periodic arrangement, where the spins exhibit long-range ordered alignment in the lattice. Magnetostats, as collective excitations, originate from the interaction and collective behavior of local magnetic moments within the material. Their behavior is similar to the wave phenomena in optical and acoustic vibrations. Magnetostatic crystals are typically composed of magnetic ions with strong exchange interactions, which form spatially symmetric arrangements within the crystal structure, allowing spin excitations to propagate through the lattice. The dynamical characteristics of magnetostats are determined by factors such as magnetic anisotropy, exchange interactions, and external magnetic fields in the crystal. The properties of magnetostatic crystals are influenced not only by the properties of the individual magnetic ion materials but also by the crystal structure and external conditions. With the in-depth study of the structures of magnetic

materials, the spin-wave modes in magnetostatic crystals exhibit strong quantum effects, providing a new theoretical foundation for applications in spintronics and quantum computing <sup>[1]</sup>.

### ***1.2 Concept and Dynamics of Spin Waves***

Spin waves are collective excitations of the spin system in magnetic materials, caused by the propagating phenomenon induced by the wave-like arrangement of local spins. In magnetic crystals, when local spins deviate from their equilibrium positions, the interaction of adjacent spins leads to the propagation of spin waves. Spin waves typically propagate in the form of waves within the lattice, with their propagation speed closely related to factors such as the material's exchange coupling strength, the geometric structure of the crystal, and external magnetic fields. The dynamics of spin waves can be described by classical spin equations or quantum spin Hamiltonians, involving exchange interactions, magnetic anisotropy, and spin scattering processes. In magnetostatic crystals, the propagation of spin waves depends not only on the material's magnetic properties but is also influenced by factors such as crystal defects, interface effects, and temperature variations. Moreover, as nonlinear effects and interactions among spin waves increase, mode coupling and frequency modulation phenomena may occur within the system, which is particularly prominent in magnetostatic crystals, greatly enriching their dynamical behavior <sup>[2]</sup>.

### ***1.3 Band Structure and Spin-Wave Modes***

The band structure is an important tool for describing collective excitations such as electrons, vibrations, or spins in solid materials. In magnetostatic crystals, the band structure not only involves the distribution of electronic energy states but also includes the formation and evolution of spin-wave modes. The propagation of spin waves in magnetostatic crystals manifests as wave-like modes with discrete energy spectra, with the energy difference between these modes corresponding to the bandgap. The band structure of spin waves is influenced by multiple factors, including exchange interactions, crystal symmetry, external magnetic fields, and lattice defects. In an ideal magnetostatic crystal, spin-wave modes typically exhibit a band-like structure, and as external conditions (such as magnetic field and temperature) change, the band structure may undergo significant alterations. For instance, under a strong magnetic field, spin-wave modes may experience transitions across the bandgap or undergo band reconstruction, thereby affecting the propagation characteristics and dynamic response of the spin waves. Additionally, due to the interaction between spin waves and other excitation modes, such as electrons and phonons, the band structure of magnetostatic crystals exhibits strong coupling effects, further expanding their potential for various applications in quantum information processing and spintronics.

## **2. Spin-Wave Mode Coupling Mechanism and Effects**

### ***2.1 Spin-Wave Mode Coupling Mechanism***

The coupling mechanism of spin-wave modes is an important physical phenomenon in magnetostatic crystals. The fundamental cause of this coupling lies in the interaction between spins and the symmetry of the crystal structure. The coupling of spin waves depends not only on the strength of exchange interactions but also on external magnetic fields, magnetic anisotropy, and lattice defects.

#### ***2.1.1 Exchange Interactions and Spin-Wave Coupling***

In magnetostatic crystals, the exchange interaction between adjacent magnetic moments is the fundamental mechanism driving spin-wave mode coupling. The exchange interaction transfers energy between interacting spins, allowing spin-wave modes with different wave vectors to interact within the lattice. When the exchange interaction is strong, significant coupling effects may form between spin-wave modes, manifesting as frequency mixing or phase-locking phenomena between modes <sup>[3]</sup>.

#### ***2.1.2 Magnetic Anisotropy and Spin-Wave Mode Splitting***

Magnetic anisotropy is another important factor influencing spin-wave mode coupling. Anisotropic magnetic fields in materials can cause the splitting of spin-wave modes, leading to spin-wave modes in different directions. These modes couple within the crystal, and by adjusting anisotropy parameters (such as magnetic anisotropy constants), the coupling strength between different spin-wave modes can be precisely controlled, thus tuning the propagation characteristics of spin waves.

### ***2.1.3 Nonlinear Effects and Spin-Wave Mode Interactions***

In addition to linear coupling, nonlinear effects also play an important role in the coupling of spin-wave modes. Nonlinear effects arise from the interaction forces between spins, which increase with the excitation intensity. Under nonlinear coupling, spin waves can exhibit phenomena such as dual-frequency modulation and waveform distortion, allowing spin-wave modes with different frequencies to influence each other and exchange energy.

## ***2.2 Impact of Coupling Effects on the Dynamic Behavior of Magnetostatic Crystals***

The coupling effects between spin-wave modes significantly affect the dynamic response of magnetostatic crystals, especially under multimode and nonlinear coupling conditions, where the system's response may change dramatically. These coupling effects not only influence the propagation speed of spin waves but can also lead to the emergence of new dynamic behaviors <sup>[4]</sup>.

### ***2.2.1 Multimode Coupling and Dynamical Instability***

When multiple spin-wave modes couple, the dynamic behavior of the system may exhibit characteristics of multimode excitation. This multimode coupling typically leads to dynamical instability, manifested as interference between modes and frequency mixing. Especially under strong coupling conditions, nonlinear amplification or attenuation of spin waves may occur, further affecting the propagation and stability of the spin waves.

### ***2.2.2 Control of Spin-Wave Propagation and Frequency Regulation***

The coupling effects between spin-wave modes can effectively control the propagation speed and frequency of spin waves. By adjusting coupling strength and interaction parameters, the frequency of spin waves within the crystal can be regulated, a property that is significant for spin-wave communication and information transmission. For example, under high magnetic fields or electric fields, spin-wave modes in magnetostatic crystals may exhibit frequency tunability, thereby affecting the system's resonance response and wave propagation characteristics.

### ***2.2.3 Nonlinear Interactions and Energy Transfer***

Nonlinear coupling effects make energy transfer between spin waves not limited to energy exchange between single-frequency modes. By adjusting the nonlinear strength in the system, energy transfer between spin-wave modes with different frequencies can be promoted, leading to nonlinear transmission behaviors of spin waves. This non-unidirectional energy transfer not only affects the transmission efficiency of spin waves but may also induce new collective dynamic phenomena, such as collective excitation and resonance behaviors of spin waves.

## ***2.3 Spin-Wave Mode Coupling and Band Reconstruction***

The coupling effects between spin-wave modes directly influence the band structure in magnetostatic crystals. The interaction between different spin-wave modes can lead to the reconstruction of the original band structure, thus affecting the propagation characteristics of spin waves and their dynamic behavior in magnetostatic crystals.

### ***2.3.1 Changes in Bandgap Due to Coupling***

The coupling effects between spin-wave modes can cause changes in the bandgap of magnetostatic crystals. By adjusting the coupling strength or the influence of external magnetic fields, the bandgap can be compressed or expanded, which in turn affects the transmission characteristics of spin waves. Under strong coupling conditions, the bandgap may completely vanish, leading to frequency overlap between spin-wave modes, which in turn alters the band structure.

### ***2.3.2 Nonlinear Coupling and Topological Changes in the Band Structure***

Nonlinear coupling effects not only change the energy spectrum of the band structure but may also lead to topological changes in the band structure. Under strong nonlinear effects, spin-wave modes may form new band crossings or new frequency bands within the crystal. Such topological changes affect the transmission modes of spin waves and the system's collective excitation behavior. For instance, in certain materials, nonlinear coupling can cause band inversion or reversal, thereby changing the stability and dynamic response of the system.

### ***2.3.3 Control of Band Reconstruction through Spin-Wave Mode Coupling***

Through the regulation of external fields (such as magnetic fields, electric fields, and temperature), the coupling effects of spin-wave modes can achieve precise control over the band structure. For example, an external magnetic field can not only change the frequency of spin waves but also, by adjusting coupling strength and interactions between modes, achieve local reconstruction of the band structure. This control mechanism provides new ideas for designing novel spintronic devices, especially in quantum information processing and spin-wave communication, with significant application value.

## **3. Band Structure Control and Application Prospects of Magnetophonon Crystals**

### ***3.1 Methods of Band Structure Control***

The methods of controlling the band structure in magnetophonon crystals are crucial for optimizing the propagation characteristics of spin waves. By using external magnetic fields, structural design, and nonlinear effects, the band structure of spin waves can be precisely adjusted to meet different application requirements [5].

#### ***3.1.1 External Magnetic Field Control***

External magnetic fields directly affect the spin wave modes in magnetophonon crystals. Under the influence of an applied magnetic field, the band structure of the crystal is adjusted, resulting in frequency shifts and changes in the band gap of the spin wave modes. The intensity and direction of the external magnetic field can alter the propagation speed of spin waves and may lead to the creation of band crossing points or modifications in the band gap. By precisely controlling the external magnetic field, the band structure can be finely tuned, enabling efficient information transmission and processing in spin-wave communication and information technologies.

#### ***3.1.2 Crystal Structure and Geometric Design***

The geometric structure and lattice arrangement of magnetophonon crystals play a critical role in band structure control. By adjusting the size, shape, defects, and doping elements of the crystal, the propagation behavior of spin waves can be modulated in localized regions, thereby influencing the overall band structure. For example, geometric control at the nanoscale can introduce local magnetic field gradients within the magnetophonon crystal, altering the propagation path and speed of spin wave modes, and potentially even triggering band reconstruction. The design of the crystal shape and microstructure optimization provide new degrees of freedom for spin-wave transmission and band structure control.

#### ***3.1.3 Nonlinear Effects and Coupling Control***

Nonlinear effects play an important role in the coupling and band reconstruction of spin waves. By adjusting the coupling strength between different spin wave modes, nonlinear effects can lead to frequency modulation and dynamic reconstruction of the band structure. Under strong coupling conditions, interactions between different spin wave modes can induce frequency mixing or energy exchange, further affecting the band structure. Nonlinear effects, particularly through dual-frequency modulation and nonlinear interactions, can further optimize the band characteristics of magnetophonon crystals, enabling more efficient energy transmission and control.

### ***3.2 Applications of Magnetophonon Crystals in Spintronics***

The control of spin wave modes in magnetophonon crystals provides a new material foundation for the field of spintronics. Spintronics utilizes the electronic spin degree of freedom for information processing and storage, and the unique properties of magnetophonon crystals show great potential in several cutting-edge applications [6].

#### ***3.2.1 Spin-Wave Communication and Information Transmission***

Magnetophonon crystals have significant applications in spin-wave communication. By precisely controlling the propagation characteristics of spin waves, efficient and low-power information transmission can be achieved. Spin waves in magnetophonon crystals propagate with lower energy loss compared to traditional electronic signals and can transmit information over longer distances. The band structure control of magnetophonon crystals enables modulation, amplification, and transmission of

spin-wave signals, driving the development of spin-wave communication technology, especially in quantum communication and information storage.

### ***3.2.2 Spin-Wave Logic Computation***

Spin wave modes in magnetophonon crystals can be used as carriers for logic operations. In spin-wave logic computation, different logical operations can be performed by adjusting the frequency, phase, and coupling strength of spin waves. For instance, the spin wave modes in magnetophonon crystals can act as switch elements, controlling the propagation state of spin waves through nonlinear coupling effects, thus completing basic logical operations. Spin-wave logic computation not only offers lower power consumption but can also operate at higher frequencies, providing greater computational efficiency compared to traditional electronic computing.

### ***3.2.3 Spin-Current Control***

The application of magnetophonon crystals in spin-current control has also attracted widespread attention. Spin wave modes can interact with currents, and by controlling the transmission and reflection of spin currents, the modulation of spin waves can be achieved. By adjusting the band structure of magnetophonon crystals, the direction and intensity of spin currents can be controlled, thus enabling efficient spin-wave transmission and signal processing. This property makes magnetophonon crystals crucial for spin-current control, storage, and processing, particularly in the development of future spintronic devices, where they will play a key role.

## ***3.3 Application Prospects of Magnetophonon Crystals***

As a novel magnetic material, magnetophonon crystals have broad application prospects in various fields. Their ability to control spin-wave modes and band structures offers new technological platforms for quantum information processing, spin-wave computation, sensor technologies, and more.

### ***3.3.1 Quantum Information and Quantum Computing***

Magnetophonon crystals have particularly broad prospects in the field of quantum information processing. By controlling the band structure of spin-wave modes, magnetophonon crystals can play a crucial role in quantum bit storage, quantum information transmission, and the implementation of quantum logic gates. The low-loss propagation of spin waves provides a significant advantage in quantum bit transmission and control for quantum computers. Furthermore, the nonlinear control capabilities of magnetophonon crystals offer new possibilities for precision control in quantum computing, especially in the realization of quantum computing networks, where their tunable band structures can significantly enhance the efficiency and stability of quantum information transmission.

### ***3.3.2 Spin-Wave Computation and New Computing Architectures***

The applications of magnetophonon crystals are not limited to quantum information; they also hold broad prospects in spin-wave computation. Spin-wave logic computation, based on the propagation and coupling of spin waves in magnetic materials, can achieve low-power, high-efficiency information processing. Magnetophonon crystals provide an ideal material foundation for spin-wave computation, and by precisely adjusting their band structure, new spin-wave computation architectures can be constructed. This has potential application value in future fields of computer science and artificial intelligence. The advantages of spin-wave computation include low energy consumption, high speed, and the ability for large-scale parallel computation, offering revolutionary solutions for future high-performance computing.

### ***3.3.3 High-Precision Sensors and Detectors***

Magnetophonon crystals also have great prospects in high-precision sensors and detectors. The sensitivity of spin waves to external factors such as magnetic fields, electric fields, and temperature enables magnetophonon crystals to be used for precise magnetic field sensing, temperature measurement, and electric field detection. By leveraging their unique band structure control capabilities, magnetophonon crystals can achieve higher sensitivity and resolution in high-precision sensors. These sensors have broad applications in quantum sensing, precision measurement, and nanotechnology, especially in fields such as healthcare, national defense, and materials science, where magnetophonon crystals will provide essential technological support.

## Conclusion

This paper investigates the spin wave mode coupling mechanisms and band structure control in magnetophonon crystals, revealing their dynamic behavior and application potential under multi-mode coupling, nonlinear effects, and external field control. By adjusting the coupling strength of spin waves and nonlinear effects, precise control over spin wave propagation can be achieved, thereby optimizing the band structure and expanding the application prospects in spintronics and quantum information fields. Future research should further explore how to leverage the nonlinear control capabilities of magnetophonon crystals to enable efficient spin wave communication and quantum computing, particularly in quantum networks and spin wave logic computation. Additionally, with the advancement of materials science and technology, the geometric design and microstructure optimization of magnetophonon crystals will provide broader technological space for the development of novel spintronic devices, promoting the transition of spin wave technology to practical applications.

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